**General Relativity Geometry**

Recall from the last file that for a given metric gαβ, the physical increments corresponding to any book-keeping increments are:



And for what it’s worth, the proper time at any given spatial coordinate is just the physical time increment corresponding to no spatial coordinate displacement, i.e., dτ = d0/c. Let’s do a couple examples, mainly from SR actually.

**Example: Finding physical coordinates in a translating Galilean SR frame**

Consider a ‘stationary’ reference frame in empty space with metric η, and then one traveling with speed **v** w/r to it. Define the coordinate transformation to the moving frame as:

 

Let’s construct the metric in the moving frame. Should be given by:



Well, Xaα´ is the inverse transformation matrix,

 

So proceeding,



According to our formulas above, what are the proper time, physical time and distance? Well these are:



This works out to:



We can define incremental physical coordinates:



And these physical coordinates should diagonalize the metric, since in terms of them, we’d have:



Observe d´is an exact differential, so we can integrate these coordinates to define global physical coordinates,



And let’s look now at how these physical coordinates relate to the prior coordinates in the stationary reference frame. Recalling,



we have:



In matrix form, this reads:



which we’ll recognize of course as the Lorentz transformation. And it follows that the physical coordinates would diagonalize (Minkowski-ize) the metric. So lesson is that we can make any coordinate transformation we want in SR or GR, but we identify the physical coordinates the same way. And the Lorentz transformation is special because it’s a transformation from physical coordinates to physical coordinates.

**Question**

Suppose that in our primed coordinate system, a particle has the trajectory:



If the particle travels this trajectory from t´ = t´1 to t´ = t´2, what will be the physical elapsed time in the primed frame? Well, apropos the physical time in the primed frame, using,



we have:



What will be the elapsed time from the particle’s perspective?

**Example: Finding physical coordinates in a rotating Galilean SR frame**

What does the SR metric look like in a rotating reference frame? What are the physical coordinates? Consider a ‘stationary’ reference frame in empty space, and then one rotating with angular velocity ω w/r to it. In the stationary frame, the line element should be,



In matrix form, this corresponds to the following metric:



Define the coordinate transformation to the moving frame as:

 

Let’s construct the metric in the moving frame. Should be given by:



Well, Xaα´ is the inverse transformation matrix,

 

So proceeding,



This corresponds to a line-element:



According to our formulas above, what are the proper time, physical time and distance? Well these are:



Can verify that the line element ds2 = -cd´2 +d´2 does indeed reduce to what we calculated via the matrix product just above. If we define vθ = ωr, and βθ = ωr/c, and γθ = 1/√(1-βθ2), then we can write these as:



We can define physical coordinate differentials:



Recall βθ and γθ depend on r´. And these would diagonalize (Minkowski-ize) the metric, locally.

We can clearly see contraction along the θ direction. Can we integrate these equations to define global physical coordinates? I think so. We can integrate the θ´ equation I’m sure since r´ would be constant along the integration.



Let’s look now at how these physical coordinates relate to the prior coordinates in the stationary reference frame. Recalling,



we have:



Not sure about this.